Rethinking Shielding Theory

David A. Larrabee
Physics Department
East Stroudsburg University
East Stroudsburg, Pa USA
dlarrabee@po-box.esu.edu

Abstract—**The traditional "shielding theory" for the shielding effectiveness of enclosures is examined. It is shown that there can be significant effects which are not included within a simple barrier based theory at high frequencies. It is shown that a 1-dimensional box (using the full Maxwell's equations solution) has 1-D resonances that can significantly affect the accuracy of a barrier based shielding calculation.**

Treating the enclosure as a resonant cavity with perturbations on the walls is a better approach to the analysis of radiated emissions and radiated susceptibility problems at high frequencies (at and above the first resonance). This has the added advantage of having been worked out quite thoroughly for wave guides and resonant cavities.

Keywords-shielding theory; radiated susceptibiliry; radiated emissions; shielding theory; resonances.

I. INTRODUCTION

Conductive enclosures are used to help shield electronics from the effects of external electromagnetic fields (radiated susceptibility) as well as to help prevent the radiated electromagnetic fields from affecting other devices (radiated emissions). The calculations and measurements of the effectiveness of these enclosures are fundamental to EMC design. Most texts in EMC spend considerable time on these subjects, for example [1], [2].

There has been ongoing interest in determining the shielding effectiveness of enclosures (both theoretically and experimentally). For instance; IEEE standards committee 299.1 has been formed to consider the shielding effectiveness of enclosures of less than 2 meters and a TC-4 working group has been established to examine the status of existing "shielding theory."

Part II examines the basis of the traditional one dimensional approach to shielding theory. Part III shows how this theory fails to account for system resonances even in one dimension. Part IV examines how the existing theory is extended into more than one dimension. Finally part V discusses the implications of these results for the practicing EMC engineer.

It is useful to explore what exactly "shielding theory" is supposed to accomplish. At a minimum "Shielding Theory" should be able to determine the MAXIMUM electric and magnetic fields inside the box due to an external electromagnetic wave. This wave should be of ARBITRARY origin, orientation, polarization and phase. Rarely does one know the location, frequency, polarization and power of the interfering source during the design phase of a project.

In principle all shielding effectiveness problems are fully described by Maxwell's equations with the appropriate constitutive relationships, geometry and boundary conditions. Many realistic problems require a computational solution. The CPU time requirements can be sufficiently large as to make this approach an impractical design tool. A practical approach to shielding theory needs to be sufficiently easy to evaluate that an EMC engineer can do the calculations on a PC in a reasonable amount of time. It is important to remember that a theory that gives an incorrect answer (or an ill-defined one) is often worse than no theory at all. Such a theory might tempt you to believe the incorrect theory and abandon your experience and common sense (rarely a good thing).

II. REVIEW OF TRADITIONAL THEORY (PART 1)

The reflection and transmission of an electromagnetic wave at an interface between two materials is presented in most textbooks on electrodynamics [3],[4], and [5]. The extension to a barrier is straight forward, but algebraically more intense. In this case the wave encounters a slab of material which presents two interfaces. The solution now involves three regions of space rather than two. A solution has been obtained for normal incidence [6], [7]. This solution can be reinterpreted as the solution of a transmission line problem where the electric field is represented as the voltage on the line and the magnetic field is represented as the current on the line. The impedance of the transmission line (Z_{TL}) is given by;

$$Z_{TL} = \sqrt{\frac{\mu}{\varepsilon(1 - j\tan\delta)}} \quad (1)$$

Here μ and ε are the magnetic permeability (henries/meter) and electric permittivity (Farads/meter) of the material and tan δ is the loss tangent of the material. In the case where the losses are due to the conductivity alone we have that

$$\tan\delta = \frac{\sigma}{\omega\varepsilon} \quad (2)$$

Here σ is the conductivity (1 / ohm-meter or mhos/meter) and ω is the angular frequency $2\pi f$ where f is in Hertz.

If the material is a vacuum (or air) then $\varepsilon_0 = 10^{-9}/36\pi$, $\mu_0 = 4\pi\ 10^{-7}$ and $\sigma = 0$ to a close approximation. This results in Z_{vac} = 377 ohms. Since one or more of these parameters will change when the material changes from air to the barrier material the impedance of the transmission line will change at that interface [8]. If the barrier material has non-zero conductivity the wave will be attenuated due to ohmic losses in the barrier, and will be frequency dependent (for details see [9]).

The simplest approach to shielding theory is to model the enclosure as a slab of the appropriate thickness. Part III shows that this can be a rather poor model. Traditionally, this slab model is then modified to account for apertures and the proximity of the source. These modifications are discussed in part IV.

A. Plane waves propagation;.

Consider an isotropic, homogeneous material that obeys linear constitutive relationships;

$$D = \varepsilon E \ , \ B = \mu H \ , \ J = \sigma E. \quad (3)$$

Here D is the electric flux density (Coulombs / meter ^{2}), E is the electric field intensity (Volts / meter), B is the magnetic flux density (Webers / meter ^{2}), H is the magnetic field intensity (Amperes / meter), J is the current density (Amperes / meter^2) and ε, μ and σ have been previously defined.

A linearly polarized plane wave propagates with the electric and magnetic field perpendicular to each other and perpendicular to the direction of the wave. The propagation vector **k** points in the propagation direction $(\mathbf{E} \times \mathbf{H})$ and has a magnitude of $2\pi/\lambda$, where λ is the wavelength in meters.

B. Plane waves at an interface.

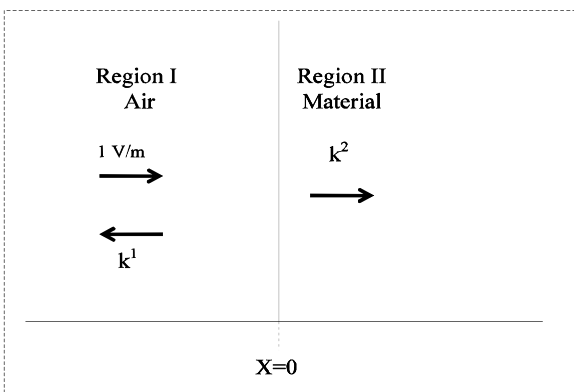

Figure 1. Nomenclature of Plane wave at an interface

Consider space to be filled with a vacuum for all points to the left of an interface (x<0) and an isotropic, homogeneous, linear material on the right of the interface (x>0) (see figure 1). A linearly polarized plane wave (E_i and H_i) impinges normally on the interface propagating from the air towards the material.

1. Some of the wave will be reflected from the interface as a linearly polarized plane wave. (E_r and H_r)
2. Some of the wave will be transmitted into the material. (E_t and H_t).

Since the material is linear all field values can be normalized to the incident field. The value of the incident field is therefore taken as 1 V/m. The ratio of the amplitude of the electric and magnetic fields of each wave is equal to the wave impedance for that medium. Therefore the incident magnetic field (H_i) has a value of 1/377 Ampere/meter. The amplitude and phase of both the transmitted and reflected electric and magnetic fields needs to be calculated.

The total tangential electric field must be the same across the boundary ($E_r+E_i=E_t$ at x=0). Similarly the total tangential magnetic field is also the same across the boundary ($H_r+H_i=H_t$ at x=0). In the case of normal incidence, both the electric and magnetic fields are parallel to the interface.

Two transmission lines of differing characteristic impedances joined at the origin have the same formal solution. This equivalence can be extended to waves whose propagation vector is not normal to the interface.

C. Wave Impedance

The concept of a "wave's impedance" is central to the current approach to shielding calculations [16]. Consider the reflection of a plane wave off a perfect conductor. Any electric field within the region II would result in an infinite current density. Therefore there can be no electric fields within region II and consequently no magnetic fields. The incident and reflected electric fields cancel on the surface.

$$E_r = -E_i \text{ and } E_t = 0 \text{ at } x=0 \quad (4)$$

The incident and reflected waves propagate in opposite directions. The magnetic fields have the same direction at the surface, and add. There is a change in the magnetic field as you cross the boundary. The magnetic field is reduced by currents flowing on the surface. (There is no ohmic loss due to these currents since there is no resistance!).

$$H_r = H_i \text{ and } H_t = 0 \quad (5)$$

Therefore we have for a solution;

$$E_i = \mathrm{Re}\left(e^{j(\omega t - kx)}\right) \quad H_i = \frac{1}{377}\mathrm{Re}\left(e^{j(\omega t - kx)}\right)$$

$$E_r = -\mathrm{Re}\left(e^{j(\omega t + kx)}\right) \quad H_r = \frac{1}{377}\mathrm{Re}\left(e^{j(\omega t + kx)}\right)$$

$$E_t = 0 \quad H_t = 0 \quad (6)$$

Here Re stands for the real part of the expression.

The ratio of E_i to H_i of the incident wave is 377 everywhere. The ratio of E_r to H_r on the reflected wave is 377 everywhere. The ratio R of of the measured fields E_i+E_r to H_i+H_r is given by;

$$R(x,t) = -377\frac{\sin(\omega t)\sin(kx)}{\cos(\omega t)\cos(kx)} \quad (7)$$

The sum of the two planes waves is a standing wave with the total electric field and the total magnetic field having their respective maxima separated by a quarter wavelength. At a given spatial location the electric and magnetic fields attain their peak values at different times. This is exactly analogous to a transmission line with a characteristic impedance of 377 ohms terminated in a short. (see ref [12] chapter 2).

Since R depends on time, it makes a poor choice for "impedance". The ratio of the complex expressions for the fields (real and imaginary parts) has no time dependence. It therefore makes a better definition of impedance. In this case the impedance becomes

$$Z(x) = -377\, j \tan(kx) \quad x \leq 0 \tag{8}$$

At the interface (x=0) there is no electric field. There is however a magnetic field at the interface and we have that Z(0) = 0.

A quarter wave back from the interface (at $x=-\lambda/4$ or $kx=-\pi/2$) the electric fields add and the magnetic fields cancel, so that $Z(-\lambda/4)$ becomes infinite! The reflected electric field has advanced 360 degrees in phase from the incident field and so they add (90 degrees to the surface 180 at the surface and 90 degrees returning). The reflected magnetic field has advanced 180 degrees from the incident magnetic field and so they subtract (90 degrees to the surface and 90 degrees back).

In between these positions at 1/8th of a wavelength back from the interface $Z(-\lambda/8)=377j$ ohms. This is equivalent to an inductive reactance.

3/8th of a wavelength back from the interface, $Z(-3\lambda/8)=-377j$. This is equivalent to a capacitive reactance.

A half wave length back we return to $Z(-\lambda/2)= 0$.

The impedance not only changes with position but assumes all possible values between x=0 and $x=-\lambda/2$! Change the barrier characteristics or the frequency of the incoming wave and the "impedance" at a fixed point in space will also change.

Thus the wave impedance is a property of the wave and not a property of the electric and magnetic fields at a point. The value of the wave impedance depends on the medium itself but it is a property of the wave. This is true even for a plane wave. This has nothing to do with "Far field" vs, "Near field" (the standing wave in the above problem goes on forever).

A closely coupled concept is the characteristic impedance of a transmission line. If a wave is traveling down a transmission line with a constant velocity we have for the incident voltage and current;

$$V_i = V_o e^{i(\omega t - kx)} \quad I_i = \frac{V_o}{R_o} e^{i(\omega t - kx)} \tag{9}$$

where R_o is the characteristic impedance of the line. Our barrier is the same as a shorted termination and the reflected wave has the form;

$$V_r = -V_o e^{i(\omega t + kx)} \quad I_r = \frac{V_o}{R_o} e^{i(\omega t + kx)} \tag{10}$$

The total voltage and current on the line is simply the sum of the incident and reflected voltages and currents on the transmission line. The characteristic impedance applies to the waves individually. Just like the reflection example above the voltage at a point divided by the current at the same point is not guaranteed to be equal to the characteristic impedance of the transmission lines.

Think of "wave impedance" as belonging to each wave individually (rather than to the field as a whole). The value of the wave impedance depends on the medium itself but it is a property of the wave.

D. Shielding of Plane waves by barriers.

Consider space to be filled with a vacuum for all points to the left of a barrier which occupies a region of space which is filled with an isotropic, homogeneous, linear material. The space to the right of the barrier is again filled with a vacuum.

Consider a linearly polarized plane wave impinging normally on this barrier from the left.

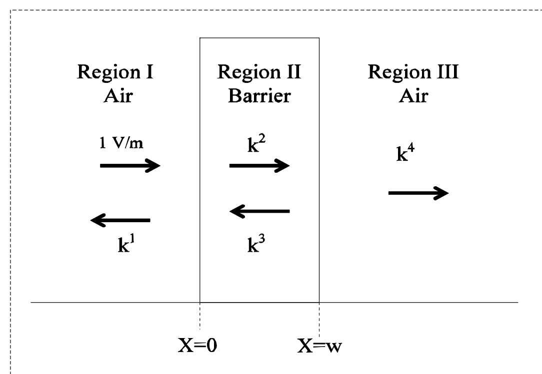

Figure 2. Nomenclature of wave penetrating a barrier

1. Some of the wave will be reflected as a linearly polarized plane wave (k^1).
2. The ratio of the electric to magnetic field in the reflected wave is 377 ohms.
3. Some of the wave will be transmitted through the barrier as a linearly polarized plane wave (k^4). The ratio of electric to magnetic field of the transmitted wave is 377.
4. Inside the barrier there will be two waves, one traveling in each direction. In each of these waves the ratio of the electric to magnetic field will be equal to the impedance of the material. This does not imply that the ratio of the total electric to magnetic field at a point in the barrier is equal to the impedance of the material (see previous section).
5. At the edges of the barrier the total tangential electric field parallel to the barrier is the same on both sides of the barrier.
6. At the edges of the barrier the total tangential magnetic field on both sides of the barrier is the same (since there are no surface currents.)

7. Since there is air on both sides of the barrier, the ratio of the electric to magnetic fields will have a ratio of 377 on both the incident and transmitted waves. This implies that the attenuation of the electric and magnetic fields by the barrier is identical.

8. This situation can also be modeled as a set of three transmission lines with the middle transmission line having the characteristic impedance of the material.

III. A PROBLEM: THE A ONE DIMENSIONAL BOX.

It is tempting to view a rectangular box as simply 6 barriers and then utilize the above theory of an infinite plane barrier. This is the approach taken in most texts on EMC shielding. The problem with this approach is that enclosures can have resonances, empty half spaces do not.

One way of seeing the limitations of the one dimensional slab model of shielding is to consider a one dimensional box. This is a "box" whose sides are infinite planes of material with a finite thickness. In radiated emissions the emitter is inside the box and we measure the fields outside the box. In radiated susceptibility we irradiate the box and consider the fields that penetrate into the interior of the box. This paper will consider the latter case.

Consider the situation shown in figure 3. There are 5 separate regions of space two of which are the sides of the box (Regions II and IV).

The electromagnetic wave is incident on the box from region I and is traveling from the left. We assume linear materials and normalize everything to the incident field. The incident electric field is therefore assigned a field strength of 1 V/m. Regions I, III and V are assumed to be a vacuum or air with an impedance of 377 ohms.

Region II and IV are barriers. Both barriers are assumed to be the same material. Two waves move in opposite directions in both barrier regions. (wave-vectors k^2, k^3 in region II and k^6, k^7 in region IV)

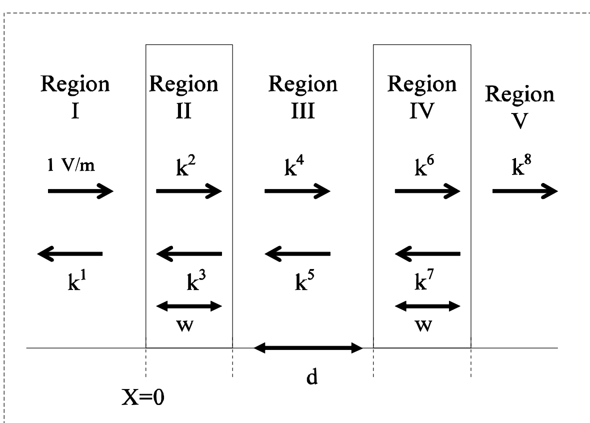

Figure 3. One dimensional box nomenclature

Region III is the inside of the box which contains air. There are two waves moving in opposite directions in this region. (wave-vectors denoted k^4, k^5)

Region V is the second "outside of the box" and contains air. There is only one wave in this region, the transmitted wave (wave-vector denoted k^8)

There are many reflections off of the interfaces. However they can all be represented by plane waves traveling in the positive directions (k^2, k^4, k^6) and in the negative direction (k^3, k^5, k^7). There is also a wave that is transmitted through the entire box (k^8).

A. Resonance

A small computer code that solves for this case as a function of frequency has been developed. It matches all the boundary conditions and yields a solution of Maxwell's equations.

Consider the case of a box with an interior dimension of 10 cm and wall thickness of 5 mm. The material of which the box is made has a relative dielectric constant and a relative permeability of 1.0. The box material is assumed to have a low conductivity of 20 mhos/meter. At each frequency the interior of the box is scanned for the peak electric field. This is plotted as a function of frequency in figure 4 (top curve). From figure 4 it is clear that there are resonances within the box. The lower dotted line represents the results of a barrier type shielding effects calculation, using a single barrier of the same material and width as the 1-D enclosure. It is clearly seen that the resonances within the box increase the electric field intensity over that of the barrier calculation. In this case at resonance the peak field amplitude is approximately 10 times the result given by the traditional barrier calculation.

The box resonates when roughly a half integer number of wavelengths fit within the box. Therefore the resonances are spaced evenly starting at $c / (2d) = 3 \times 10^8 / 0.2$ meters = 1.5 GHz where c is the speed of light and d is the inside dimensions of the box. Making the box more conductive increases the shielding effectiveness, but decreases the width of the resonance. The increase in Q comes about since less electromagnetic energy is dissipated within the box during each cycle.

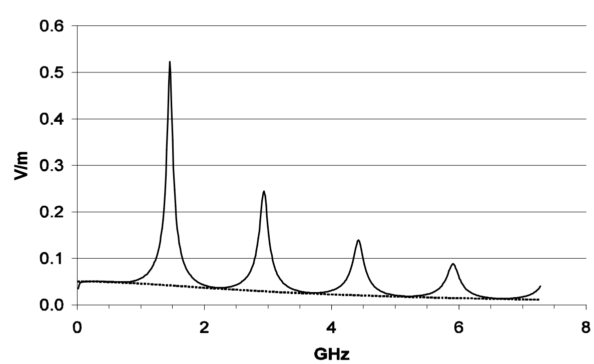

Figure 4. Resonances with resistivity of 20 mhos/m

In figure 5 the conductivity has been reduced to 2 mhos/meter. The resonant peaks are broader (lower Q) than the previous case. In addition the peak fields are now only 2.5 times the result given by the barrier calculation. Since the barrier is less conductive more of the electromagnetic field can penetrate the box. Consequently, the peak field in the box is larger than the previous case.

The barrier type shielding calculations will only yield a reliable estimate if the Q of the box resonances is low.

It is also important to remember that the value of the electric field is a function of position inside the enclosure. This especially true at resonance, where there is a predictable shape of the electric and magnetic fields inside the enclosure.

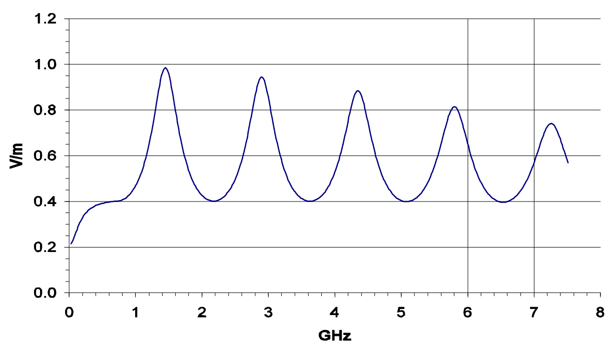

Figure 5. Maximum electric field in a 2 mhos/m box.

B. Low Frequency Limit

Figure 6 expands the low frequency region of fig 4 up to the first resonance (solid line). Again the barrier type calculation is shown as a dotted line. The region below the first resonance shows a better agreement between the barrier and box approaches to shielding calculations. Thus it appears that the barrier approach to shielding effectiveness gives reasonable estimates of shielding effectiveness at frequencies below the response of the first resonance. Notice that the Q of the first resonance will have a role in determining this upper limit.

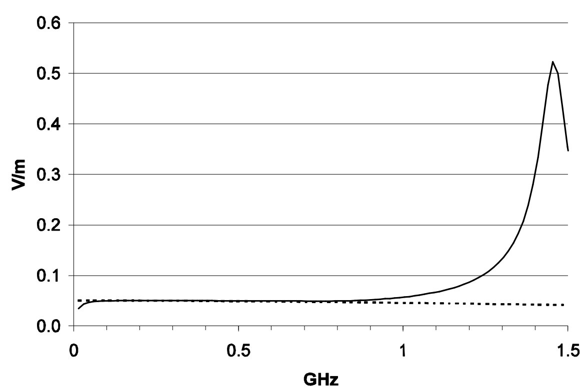

Figure 6. Low frequency behavior of the 1-D box in Figure 4.

C. Another approach to the Resonant region

The resonances of the box dominate the high frequency behavior in the resonant region. This suggests that in this region we abandon the barrier model and adopt a resonant cavity model. This model is especially well adapted to situations with thin walls of a conductive material.

The Q of the resonances is determined by the dissipation of electromagnetic energy within the box. The box shields its interior from the external magnetic field by having induced currents in the walls. The ohmic (I^2R) losses in the walls dissipate electromagnetic energy. The Q of the cavity is both a measure of the width of the resonance peak and the dissipation of electromagnetic energy in the cavity.

$$Q = f_r / \Delta f = 2 \pi E_{max} / E_{diss} \qquad (11)$$

Here f_r is the resonant frequency and Δf is the frequency difference between the half power points on the resonance curve (field intensity of ~ 0.707 of the peak). E_{max} is the maximum amount of energy that is stored in the fields during the course of 1 complete cycle. E_{diss} is the amount of electromagnetic energy dissipated in one cycle. Thus the larger amount of energy dissipated during one cycle, the lower the Q and the larger the width of the resonance. In general Q depends on material, geometry and the frequency. The resonant peaks often have different Q's.

To estimate the peak amplitude of the resonance we treat the external field as being coupled to the interior of the cavity. In this case it is coupled though "leaky" walls. It could also be coupled though holes in the wall (or cables penetrating the walls). The theory of coupling into wave guides and resonant cavity modes is well established (see [13], [14] and [15]).

None of this involves abandoning the transmission line model of the electromagnetic field (at least in one dimension). It simply means replacing the simple barrier model by a model by one that includes the resonances of the box.

IV. REVIEW OF TRADITIONAL THEORY - REALISTIC GEOMETRIES

Neither the simple barrier theory or the one dimensional box take into account that 1) Real enclosures are three dimensional, 2) the electromagnetic field may not be a plane wave and 3) the enclosures often have openings.

A. Three dimensional box

The correct solution for the three dimensional box is to utilize Maxwell's equations. The general calculation is a variation of the traditional scattering problem. If this case we consider the scattering of an electromagnetic wave that is incident on a three dimensional structure (see for instance [10]).

At frequencies where the resonances of the box are important the wavelength of the incident electromagnetic radiation is approximately the same size as the box (since the largest inside dimension of the box is about the same as the largest outside dimension of the box). The approximation that the electric field is uniform on each surface of the box is probably no longer true for near field sources. This can make the calculations much more difficult.

The resonant structure influences the scattering of the electromagnetic wave. Complicated structures still have resonances. However, those resonances have a more complex distribution than those of simple geometries. It is still expected that the maximum transfer of electromagnetic energy into the enclosure will be at the enclosure resonances. This situation is perhaps easier to think about in terms of energy. Like an antenna, the enclosure can be thought of capturing some of the energy in the wave (and reflecting some of the energy). The energy that is captured will be dissipated. Some will be lost in the metal of the enclosure itself. The rest will be dissipated within the enclosure. If the enclosure has a high Q resonance at the frequency of interest, there will be larger fields inside the enclosure.

Therefore, field intensity within the enclosure will be minimized if care is taken to make sure there is a method for absorbing RF energy within the structure (other than the electronics!). Where this is located within the structure is important as the fields will not be uniform within the enclosure.

B. Holes and penetrations in the enclosure

Holes and penetrations of the enclosure simply couple the external electromagnetic environment into the box. These coupling coefficients depend in general on the mode under consideration (as well as the geometry).

Near a hole in the enclosure the structure of the field depends of course on the shape of the hole, barrier thickness, frequency etc. But all this "structure" dies away faster than 1/ (distance from hole). These fields act to "stimulate" the resonant modes the same way that the small linear wire (or hole) in a resonator can stimulate the modes in a cavity. The details of the field geometry may not be important. A cavity model treats the wave as having a transfer impedance into the cavity. Methods for calculating these coupling coefficients are well understood (see [13], [14] and [15]).

V. CONCLUSIONS

The present barrier based transmission line based theory of shielding may yield reasonable results in the cases where

1) The frequency is below the first resonance of the box by at least several half widths of the Q of the first resonance

2) The enclosure is far enough away from the source so that the variation of the electromagnetic field intensity over the face of the box is small.

In cases where this is not true the box needs to be treated as a loaded resonant cavity. The location of the resonances and their Q need to be either calculated or measured. In addition the coupling coefficients between the external field and the interior of the box need to be determined for each of the resonances.

VI. ACKNOWLEDGEMENT

The author would like to thank the reviewer for some valuable suggestions.

REFERENCES

[1] J.L. Normal Violette, Donald R.J. White, Michael F. Violette, Electromagnetic Compatibility Handbook, New York: Van Nostrand Reinhold Co. © 1987 Chapters 10, 11, 12, pp. 310–426

[2] Henry W. Ott Noise Reduction Techniques in Electronic Systems. Second edition, New York: John Wiley & Sons 1988, Chapter 6 pp.159–202.

[3] David J. Griffiths, Introduction to Electrodynamics, New Jersey: Prentice Hall © 1999 page 382-392.

[4] Julius Adams Stratton, Electromagnetic Theorys New York: McGraw-Hill © 1941 pages 483-511.

[5] Paul Lorrain, Dale R. Corson, Fancois Lorrain, Electromagnetic Fields and Waves,Third edition New York: W.H. Freeman and Co. © 1988 page 554-573.

[6] D. S. Jones, The Theory of Electromagnetism New York: Pergamon Press © 1964 page 326-329.

[7] Julius Adams Stratton, Electromagnetic Theorys New York: McGraw-Hill © 1941 pages 511-516.

[8] Robert Grover Brown, Robert A. Sharpe, William Lewis Hughes, Robert E. Post Lines, Waves and Antennas, second edition New York: John Wiley ans Sons © 1973 page 209-220

[9] Julius Adams Stratton, Electromagnetic Theorys New York: McGraw-Hill © 1941 pages 282-284

[10] D. S. Jones, The Theory of Electromagnetism New York: Pergamon Press © 1964 Chapters 8 and 9

[11] S.A. Schelkunoff, Electromagnetic Wave New York D, Van Nostrand © 1943 pages 480-496

[12] Robert Grover Brown, Robert A. Sharpe, William Lewis Hughes, Robert E. Post Lines, Waves and Antennas, second edition New York: John Wiley ans Sons © 1973 Chapter 5

[13] Robert E. Collin, Field Theory of Guided Waves, New York, McGraw-Hill © 1960 chapter 7.

[14] N. Marcuvitz, Waveguide Handbook New York: Dover Publications © 1951 (McGraw-Hill).

[15] J. Vab Bladel, Electromagnetic Fields, New York: Hemisphere Publishing Company © 1985 chapters 10 and 11.

[16] S. A. Schelkunoff,"The Impedance Concept and Its' Application to Problems of Reflection, Refraction, Shielding and Power Absorption" Bell System Technical Journal Vol 17 No 1 Jan 1938 pgs 17-48